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A remark on the large difference between the glueball mass and T_c in quenched QCD

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Abstract. The lattice QCD studies indicate that the critical temperature $T_c \simeq 260$–280 MeV of the deconfinement phase transition in quenched QCD is considerably smaller than the lowest-lying glueball mass $m_G \simeq 1500$–1700 MeV, *i.e.*, $T_c \ll m_G$. As a consequence of this large difference, the thermal excitation of the glueball in the confinement phase is strongly suppressed by the statistical factor $e^{-m_G/T_c} \simeq 0.00207$ even near $T \simeq T_c$. We consider its physical implication, and argue the abnormal feature of the deconfinement phase transition in quenched QCD from the statistical viewpoint. To appreciate this, we demonstrate a statistical argument of the QCD phase transition using the recent lattice QCD data. From the phenomenological relation between T_c and the glueball mass, the deconfinement transition is found to take place in quenched QCD before a reasonable amount of glueballs is thermally excited. In this way, quenched QCD reveals a question "what is the trigger of the deconfinement phase transition ?"

PACS. 12.38.Mh Quark-gluon plasma – 12.38.Gc Lattice QCD calculations – 12.39.Ba Bag model – 12.39.Mk Glueball and nonstandard multi-quark/gluon states

1 Introduction

The quark-gluon plasma (QGP) is one of the most interesting targets in the finite-temperature quark-hadron physics [1]. Currently, a QGP creation experiment is being performed in the relativistic heavy-ion collider (RHIC) project at the Brookhaven National Laboratory (BNL), and much progress in understanding the finite-temperature QCD is desired. Historically, the instability of the hadron phase was first argued by Hagedorn [2] before the discovery of QCD. He pointed out the possibility of a phase transition at finite temperature, based on the string or the flux-tube picture of hadrons [2,3]. After QCD was established as the fundamental theory of the strong interaction, this phase transition was recognized as the deconfinement phase transition to the QGP phase, where quarks and gluons are liberated with the restored chiral symmetry. The QCD phase transition has been studied using various QCD-motivated effective models such as the linear σ model [4], the Nambu–Jona-Lasinio model [5], the dual Ginzburg-Landau theory [6] and so on.

In order to study nonperturbative features of the QCD phase transition, the lattice QCD Monte Carlo calculation serves as a powerful tool directly based on QCD. It has been already extensively used to study the nature of the QCD phase transition. At the quenched level, $SU(3)$ lattice QCD indicates the existence of the deconfinement phase transition of a weak first order at $T_c \simeq 260$–280 MeV [7]. On the other hand, in the presence of dynamical quarks, it indicates the chiral phase transition at $T_c = 173(3)$ MeV for $N_f = 2$ and $T_c = 154(8)$ MeV for $N_f = 3$ in the chiral limit [8].

For the comparison with the experimental data in the real world, it would be desirable to investigate full QCD with dynamical quarks. However, there are a number of underlying nonperturbative features, which are shared in common by both full QCD and quenched QCD such as color confinement and instanton phenomena. In order to understand such nonperturbative features of QCD, quenched QCD provides us with an idealized environment to focus on the essence of the problem itself without involving inessential technical complexities.

The aim of this paper is to point out the abnormal feature of the large difference between the lowest-lying glueball mass $m_G = 1500$–1700 MeV and the critical

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temperature $T_c = 260$–280 MeV in quenched QCD. In fact, due to this large difference, the thermal excitation of the glueball is strongly suppressed by a small statistical factor $e^{-m_G/T_c} \simeq 0.00207$ even near $T \simeq T_c$. In order to appreciate this point, we demonstrate a statistical argument on the QCD phase transition using the closed-packing model with the bag-model picture of hadrons. The closed-packing model has been often used for the phenomenological understanding of the full-QCD phase transition near T_c from below. In the confinement phase, only color-singlet states such as hadrons can contribute to the Boltzmann sum in the partition function, and therefore it is desirable to understand the QCD phase transition in terms of the hadronic degrees of freedom below T_c.

The contents are organized as follows. In sect. 2, we give a brief review of the closed-packing model, *i.e.*, the statistical treatment of the QCD phase transition with the bag-model picture of hadrons. We derive a phenomenological relation among the hadron masses, the hadron sizes and the critical temperature. In sect. 3, we apply the statistical approach to full QCD and compare it with the recent lattice data of full QCD. In sect. 4, we apply the statistical approach to quenched QCD, and demonstrate an abnormal feature of the quenched-QCD phase transition. In sect. 5, after the summary of the results, we attempt to clarify the essence of this problem, and discuss the abnormal nature of the deconfinement phase transition in quenched QCD.

2 A review of the statistical approach

In this section, we give a brief review of the closed-packing model, *i.e.*, the statistical treatment of the QCD phase transition based on the bag-model picture of hadrons. It provides us with a simple but useful insight into the QCD phase transition from below T_c in terms of hadronic properties. We derive a phenomenological relation between the critical temperature T_c and the properties of hadrons, *i.e.*, mass and size.

In the bag-model picture, quarks and gluons are assumed to be confined inside a spherical bag. Color confinement is simply taken into account through the bag-like intrinsic structure of hadrons [9]. At low temperature, only a small number of such bags are thermally excited, and the thermodynamic properties of the system are described in terms of these spatially isolated bags. With increasing temperature, the number of the thermally excited bags increases. Gradually, these bags begin to overlap one another, and they finally cover the whole space region at a critical temperature T_c in this picture. Above T_c, as a result of overlapping bags, the whole space is filled with liberated quarks and gluons, and the thermodynamic properties of the system are now governed by quarks and gluons. In this way, the QCD phase transition is described in terms of the overlaps of the thermally excited bags.

For the quantitative discussion, we define the spatial occupation ratio $r_V(T)$ at temperature T to be the ratio of the total volume of the spatial regions inside the thermally excited bags to the volume V of the whole space region. In

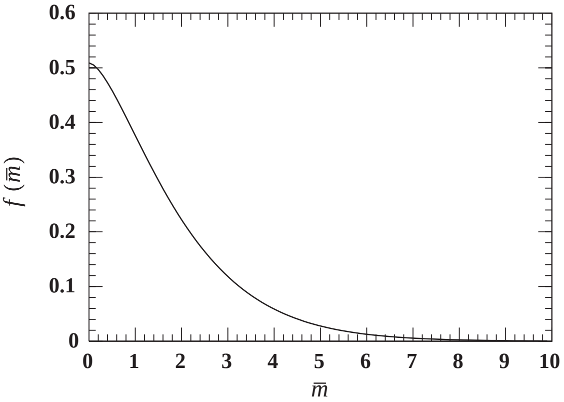

Fig. 1. The characteristic function $f(\bar{m})$ of the thermal boson in eq. (2) plotted against $\bar{m} = m/T$.

the closed-packing picture of the QCD phase transition, $r_V(T)$ plays a key role, and is estimated as

$$r_V(T) = \frac{1}{V}\sum_n \frac{4\pi}{3}R_n^3 \cdot \lambda_n N_n(T) \tag{1}$$
$$= \sum_n \lambda_n R_n^3 \frac{4\pi}{3}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{e^{\sqrt{m_n^2+\mathbf{k}^2}/T}-1}$$
$$= \sum_n \lambda_n R_n^3 T^3 f(m_n/T),$$

where $N_n(T)$, λ_n, m_n and R_n are the number at temperature T, the degeneracy, the mass and the bag radius of the n-th elementary excitation, respectively. Here, $f(\bar{m})$ is defined by

$$f(\bar{m}) \equiv \frac{4\pi}{3}\int \frac{\mathrm{d}^3\bar{k}}{(2\pi)^3}\frac{1}{e^{\sqrt{\bar{m}^2+\bar{k}^2}}-1}, \tag{2}$$

and its functional form is plotted against $\bar{m} \equiv m/T$ in fig. 1. Note that $f(m/T)$ is a characteristic function to describe the thermal contribution of a boson with mass m at temperature T [10]. For $m \gg T$, $f(m/T)$ decreases exponentially with m/T, and the thermal contribution is expected to become negligible.

In the closed-packing picture, the phase transition is assumed to take place when the thermally excited bags almost cover the whole space region. Hence, the critical temperature T_c is estimated by solving

$$r_V(T_c) = 1. \tag{3}$$

In the closed-packing picture, an essential role is played by the color confinement, which is a peculiar phenomenon in QCD making the underlying quark and gluon structure hidden inside the hadron. This feature is quite different from the atomic system. (For instance, the closed-packing picture of the phase transition cannot be applied to the Coulombic system such as the ionization transition of an atomic gas, where the bag-model description of the bound state is inappropriate.) The readers might feel that the closed-packing argument is too naive for the complicated QCD phase transition. In particular, the ideal-gas

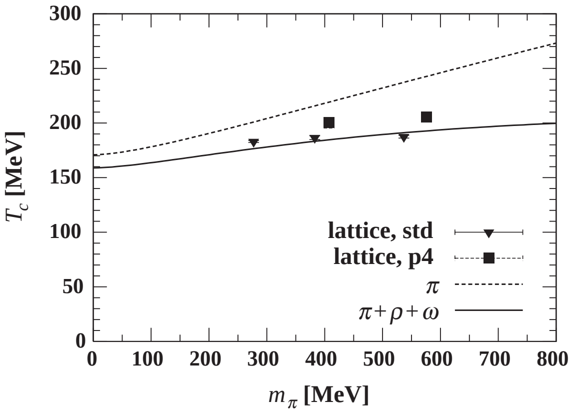

Fig. 2. The critical temperature T_c plotted against the pion mass m_π in the $N_f = 2$ case. The dashed curve denotes the result retaining only the contribution from pions. The solid curve denotes the result including also contributions from ρ- and ω-mesons. The triangle denotes the lattice data taken from ref. [11] obtained with standard (std) fermion action. The square denotes the lattice data taken from ref. [8] obtained with the improved staggered (p4) fermion action.

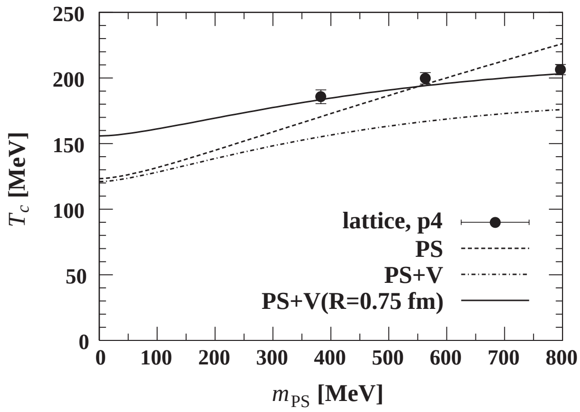

Fig. 3. The critical temperature T_c plotted against the pseudo-scalar meson mass $m_{\rm PS}$ in the $SU(3)_f$ symmetric case. The dashed curve denotes the result retaining only the contribution of the pseudo-scalar mesons. The dot-dashed and the solid curves denote the results including also the contribution of the flavor-octet vector mesons with $R = 1$ fm and $R = 0.75$ fm, respectively. The circle denotes the lattice data taken from ref. [8] obtained with the improved staggered (p4) fermion action.

statistical treatment should be applied only to the dilute system in a strict sense. Nevertheless, the closed-packing approach provides a simple and physical insight into the complicated QCD phase transition in terms of the thermal excitations of hadrons. Hence, it would be natural to attempt to understand the QCD phase transition in the closed-packing picture as a first step.

3 A statistical approach to the full-QCD phase transition

First, we apply the statistical approach with the closed-packing model to the full-QCD phase transition, and compare the results with the recent lattice QCD data. In full QCD, the lightest physical excitation is the pion, and all the other hadrons are rather heavy: $m \gg m_\pi$, T_c. In fact, the pion is considered to play the key role in describing the thermodynamic properties of full QCD below T_c from the viewpoint of the statistical physics. Hence, in most cases, only the pionic degrees of freedom are taken into account in the hadron phase in the argument of the full-QCD phase transition. By using the isospin degeneracy $\lambda_\pi = 3$, the mass $m_\pi = 140$ MeV and the radius $R_\pi \simeq 1$ fm, we solve eq. (3) with $r_V(T) = 3R_\pi^3 T^3 f(m_\pi/T)$ to estimate the critical temperature as $T_c \simeq 183$ MeV. Considering its closeness to the full lattice QCD result with $N_f = 2$, *i.e.*, $T_c \simeq 170$ MeV, the statistical approach to the full-QCD phase transition seems to be rather good.

We now consider the m_π-dependence of the critical temperature T_c. Note that, in the actual lattice QCD calculations, the pion mass is taken to be still rather heavy *i.e.* $m_\pi \gtrsim 400$ MeV for technical reasons. From these data, the critical temperature T_c in the chiral limit is obtained using the chiral extrapolation. In ref. [8], the authors parametrized the m_π-dependence of T_c in the full lattice QCD with $N_f = 2$ as

$$\left(\frac{T_c}{\sqrt{\sigma}}\right)(m_\pi) = 0.40(1) + 0.039(4)\left(\frac{m_\pi}{\sqrt{\sigma}}\right), \qquad (4)$$

where σ denotes the string tension. Strictly speaking, the phase transition becomes just a cross-over for intermediate values of m_π. Hence, in ref. [8], the pseudo-critical temperature is adopted as T_c, which is determined from the peak positions of the susceptibilities of the Polyakov loop and so on.

For the $N_f = 2$ case, fig. 2 shows the estimate of T_c plotted against m_π based on eq. (2) in the closed-packing model. The dashed curve denotes T_c retaining only the contribution from pions with $R_\pi \simeq 1$ fm. The solid curve denotes the result in the case including also the contributions from the low-lying vector mesons, such as ρ and ω, which are the next lightest particles in $N_f = 2$ full QCD. Here, we have used $\lambda_\rho = 3 \times 3 = 9$, $m_\rho = 770$ MeV, $R_\rho \simeq 1$ fm, $\lambda_\omega = 3$, $m_\omega = 783$ MeV, $R_\omega \simeq 1$ fm as inputs, which are treated as m_π-independent constants. These vector mesons give an additional contribution to $r_V(T)$ as $\delta r_V(T) = 9R_\rho^3 T^3 f(m_\rho/T) + 3R_\omega^3 T^3 f(m_\omega/T)$. The triangle and the square in fig. 2 denote the lattice data taken from refs. [8,11]. The m_π-dependence of the critical temperature T_c is improved after the inclusion of the low-lying vector mesons, *i.e.*, ρ and ω. In spite of the naive-mindedness of the closed-packing model, this statistical analysis seems to reproduce the lattice data, which may be surprising.

We next consider the idealized $SU(3)_f$ symmetric case. In this case, pions, kaons and η_8 are the lightest, and form the pseudo-scalar (PS) meson octet ($\lambda_{\rm PS} = 8$), possessing the common mass $m_{\rm PS}$. We plot, in fig. 3, the critical temperature T_c against $m_{\rm PS}$. The dashed curve denotes

T_c retaining only the contribution from PS-mesons with $R_{PS} \simeq 1$ fm, which leads to $r_V(T) = 8R_{PS}^3 T^3 f(m_{PS}/T)$. The dot-dashed curve in fig. 3 denotes the results including also the contributions from the octet vector mesons with $\lambda_V = 8\times3 = 24$, $m_V \simeq 770$ MeV, $R_V \simeq 1$ fm. (Inclusion of the flavor-singlet vector meson does not change the result so much.) These vector mesons give an additional contribution to $r_V(T)$ as $\delta r_V(T) = 24R_V^3 T^3 f(m_V/T)$. The circle in fig. 3 denotes the lattice data taken from ref. [8]. We see that both the dashed and the dot-dashed curves are roughly consistent with the lattice QCD results. We note that the small deviation almost disappears by slightly adjusting the bag size as $R_{PS} = R_V = 0.75$ fm, as is shown in fig. 3 by the solid curve.

Thus, the simple statistical approach with the closed-packing picture is seen to reproduce the recent lattice QCD data on the critical temperature T_c and its m_{PS}-dependence in the full-QCD phase transition both for $N_f = 2$ and $N_f = 3$, which would suggest that some of essential natures of the full-QCD phase transition could be governed by low-lying hadrons. Considering its naivete and its simple-minded nature, this coincidence may be surprising. It would be interesting to refine this approach by including the interaction among hadrons. However, we emphasize that the application of the closed-packing model to full QCD is not the final aim of this paper. Rather, our aim is to demonstrate the mysterious mismatch between the critical temperature and the mass of the elementary excitation mode in the quenched-QCD phase transition in the next section.

4 A mystery in the quenched-QCD phase transition

In this section, we come to the main point of the paper, *i.e.*, the crucial mismatch of the critical temperature T_c and the low-lying elementary-excitation mode in the quenched-QCD phase transition. Focusing on the large difference between the lowest-lying glueball mass $m_G \simeq 1.5$–1.7 GeV and $T_c \simeq 280$ MeV in quenched QCD, we consider its physical implications. To make the argument more quantitative, we combine the recent lattice QCD data with the statistical approach, and attempt to make an estimate of the critical temperature T_c.

In quenched QCD, due to the color confinement, only the color-singlet modes can appear as physical excitations. Since all of them consist of color singlet combinations of gluons, they are called glueballs. The mass spectrum of the glueballs is known through the quenched lattice QCD calculations [12–14]. The lightest physical excitation is the 0^{++} glueball with $m_G = 1.5$–1.7 GeV. Being the lightest physical excitation, the lowest 0^{++} glueball is expected to play a key role in describing the thermodynamic properties of quenched QCD in the confinement phase. Hence, we first take into account only the 0^{++} glueball. For the mass $m_{G(S)}$ and the size $R_{G(S)}$ of the lowest 0^{++} glueball, we adopt the recent lattice QCD results: $m_{G(S)} = 1730$ MeV [12] and $R_{G(S)} = 0.4$ fm [15,16]. We use these values

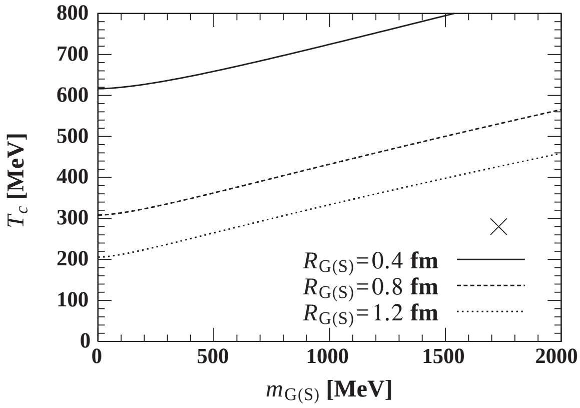

Fig. 4. The critical temperature T_c plotted against the lowest 0^{++} glueball mass $m_{G(S)}$ in the statistical approach. The solid line denotes T_c for the glueball size $R_{G(S)} = 0.4$ fm, which is taken from the recent lattice QCD [15,16]. The cross ($\times$) indicates the quenched lattice QCD results, *i.e.*, $T_c = 280$ MeV and $m_{G(S)} = 1730$ MeV. We add the cases with $R_{G(S)} = 0.8$ and 1.2 fm denoted by the dashed and dotted lines, respectively.

together with the degeneracy $\lambda_{G(S)} = 1$ as inputs. Then the spatial occupation ratio is given by $r_V(T) = R_{G(S)}^3\, T^3\, f(m_{G(S)}/T)$, and the critical temperature is estimated as $T_c \simeq 827$ MeV from eq. (3). This estimate is much larger than the quenched lattice QCD result $T_c = 0.629(3)\sqrt{\sigma} \simeq 280$ MeV of ref. [7] with $\sqrt{\sigma} = 450$ MeV. (In other words, only a tiny fraction of the space region can be covered by the thermally excited bags: $r_V(T) = 0.0021$ at $T = 280$ MeV.) This large discrepancy would be a serious problem in the quenched-QCD phase transition.

To seek the solution, we examine the possibilities of the pole-mass reduction and the thermal swelling of the glueball at $T \simeq T_c$, which are suggested in ref. [6]. We first consider the possibility of the pole-mass reduction at $T \simeq T_c$. To this end, we show in fig. 4 the critical temperature T_c plotted against the lightest 0^{++} glueball mass $m_{G(S)}$ in the closed-packing model for the glueball size $R_{G(S)} = 0.4$ fm with the solid curve. The cross ($\times$) indicates the quenched lattice QCD results, $T_c = 280$ MeV and $m_G = 1730$ MeV. We see that, for the problem to be settled, the pole-mass reduction must be as significant as $m_{G(S)}(T_c) \lesssim 500$ MeV. However, in the recent lattice QCD calculations [15,16], it has been reported that the thermal 0^{++} glueball persists to hold a rather large pole mass: $m_{G(S)}(T \simeq T_c) \simeq 1250$ MeV. Hence, we examine another possibility, *i.e.*, the thermal swelling. In fig. 4, we also include T_c corresponding to different sizes of the glueball, *i.e.* $R_{G(S)} = 0.8, 1.2$ fm, which are denoted by the dashed and dotted curves, respectively. We see that to reproduce $T_c \simeq 280$ MeV, the thermal swelling of the glueball must be quite significant at $T \simeq T_c$. The explicit calculation leads to an abnormally large size: $R_{G(S)} \simeq 3.1$ fm. However, such a drastic thermal swelling of the lowest scalar glueball was rejected by the recent lattice QCD studies [16,15], which state that the thermal glueball size is almost unchanged even near T_c.

We seek for another possibility by including the contribution of the excited-state glueballs. In addition to the lowest 0^{++} glueball with $m_{G(S)} \simeq 1730\,\mathrm{MeV}$ and $R_{G(S)} \simeq 0.4\,\mathrm{fm}$, we consider the thermal contribution from the lowest 2^{++} glueball, which is the next lightest hadron in quenched QCD. We take $\lambda_{G(T)} = 5$, $m_{G(T)} \simeq 2400\,\mathrm{MeV}$ [12], $R_{G(T)} \simeq 1\,\mathrm{fm}$. (Here, we assume it to have a typical hadron size.) Then, the spatial occupation ratio receives a correction as $\delta r_V(T) = 5R^3_{G(T)}T^3 f(m_{G(T)}/T)$, and the correction amounts to $\delta r_V(T) = 0.0223$ at $T = 280\,\mathrm{MeV}$. The resulting critical temperature is given as $T_c \simeq 432\,\mathrm{MeV}$, which is still too large. We note that the realistic glueball size would be more compact. However, if so, its contribution becomes more negligible. Besides these two low-lying glueballs, the following excited states are predicted in ref. [12]: $0^{-+}(2590)$, $0^{*++}(2670)$, $1^{+-}(2940)$, $2^{-+}(3100)$, $3^{+-}(3550)$, $0^{*-+}(3640)$, $3^{++}(3690)$, $1^{--}(3850)$, $2^{*-+}(3890)$, $2^{--}(3930)$, $3^{--}(4130)$, $2^{+-}(4140)$, $0^{+-}(4740)$. We include all the contributions from these excited states, assuming the unknown glueball size as a typical hadron size, *i.e.*, $R_G \simeq 1$ fm. The correction by these excited states amounts to only $\delta r_V(T) = 0.0113$ at $T = 280\,\mathrm{MeV}$, and the resulting critical temperature is estimated as $T_c = 395\,\mathrm{MeV}$, which is again too large. Note that we have used $R_G \simeq 1$ fm for the radii of the excited glueballs, since we adopt it as a typical hadron size. However, even if we adopt $R_G \simeq 2$ fm, which may be considered well beyond the upper bound for a single-hadron radius, the total contribution to $r_V(T)$ is $\delta r_V(T) = 0.269$ at $T = 280\,\mathrm{MeV}$, which is still insufficient.

One may argue that our statistical argument can be improved by considering the self-interaction of the glueball and the Lorentz contraction effects. However, unlike in full QCD, the glueball system at $T \simeq T_c$ is so dilute that the ideal-gas statistical treatment can be applied. (We will come back to this point in the summary.) On the other hand, due to the large difference of T_c and the glueball masses, the effect of the Lorentz contraction is negligible. Furthermore, the Lorentz contraction effect drives the discrepancy in the unwanted direction.

To recapitulate this section, we have observed that the statistical approach leads to a terrible overestimate of T_c in quenched QCD even after so many improvements have been attempted. The direct cause of this failure is the extremely small statistical factor $e^{-m_{G(S)}/T_c} \simeq 0.0021$, which strongly suppresses the excitations of the glueballs. As a consequence, only a small fraction of the space region can be covered by the thermally excited bags of glueballs even at $T = 280$ MeV, and T_c becomes abnormally large.

5 Summary and discussions —What is the trigger or the driving force of the QCD phase transition?

We have considered the large difference between the glueball mass $m_G \simeq 1500$–1700 MeV and the critical temperature $T_c \simeq 260$–280 MeV in the quenched QCD. As a consequence of this large difference, the thermal excitation of a single glueball is suppressed by a strong statistical factor $e^{-m_G/T_c} \simeq 0.00207$. We have considered its physical implications and argued about the abnormal nature of the quenched-QCD phase transition. To appreciate how abnormal it is, we have used the statistical argument with the bag-model picture of hadrons, *i.e.*, the closed-packing model. We have derived a phenomenological relation among the critical temperature T_c, the mass and the size of the low-lying hadrons.We have demonstrated that with slight modifications of the model parameters, the closed-packing model can reproduce the m_π-dependence of T_c obtained by full lattice QCD for the $N_f = 2$ and 3 cases, suggesting that some of the essential ingredients of the full-QCD phase transition is governed by the low-lying hadrons.

Unlike the full QCD, we have found that the statistical approach terribly overestimates the critical temperature $T_c \simeq 827$ MeV in quenched QCD, and that only a tiny fraction of the space region is covered by the thermally excited bags of glueballs at $T \simeq 280$MeV. We have considered the possibility of the thermal swelling of the glueball size and the pole-mass reduction of the glueball near the critical temperature. However, both of these two possibilities have not provided us with the solution to explain this large discrepancy. Even though one includes all the contributions from the 15 low-lying glueballs up to 5 GeV predicted in quenched lattice QCD [12], the discrepancy is still large. In other words, the number of the thermally excited glueballs is still too small at $T \simeq 280$MeV, even after so many glueball excited states are taken into account. The direct origin of this discrepancy is the strong suppression of the thermal excitation of glueballs due to the extremely small statistical factor $e^{-m_{G(S)}/T} = 0.00207$ at $T = 280$ MeV even for the lightest glueball, which leads to an insufficient amount of the space covered by the thermally excited bags of glueballs. We remark that the crucial role is played by the smallness of the statistical factor $e^{-m_{G(S)}/T}$, which has a rather general nature. Hence, through this failure, the closed-packing argument suggests a general tendency that, if based on the low-lying excitation modes, the natural critical temperature T_c tends to become much larger in quenched QCD. Actually, such a tendency is also found in a field-theoretical model such as the dual Ginzburg-Landau theory [6], which is an effective theory of color confinement based on the dual Meissner effect. It is remarkable that the essential origin of this large deviation of T_c can be understood in our naive statistical approach in a simplified and idealized manner.

Although several arguments given so far have been based on the bag-model picture, this problem itself has a quite general nature, which can go beyond the reliability of the model framework. Finally, we reformulate this problem in a model-independent general manner by considering the inter-particle distance l instead of the bag size R. Note that the reliability of the statistical argument improves in the dilute glueball gas limit. Now, taking into account all the low-lying 15 glueball modes up to 5 GeV predicted in quenched lattice QCD [12], we calculate the inter-particle distance l of the glueballs based only on the statistical argument. At $T = 280$ MeV, the inter-particle distance is estimated as $l \simeq 5$ fm. It follows that the deconfinement phase transition takes place at $T \simeq 280$ MeV, where the glueball density is rather small $\rho = 1/\{\frac{4\pi}{3}(2.5 \text{ fm})^3\} \simeq 1/(4.0 \text{ fm})^3$. Since the theoretical estimates of the glueball size $R_{G(S)} \lesssim 0.4$ fm are rather small [15–17], the glueball system can be regarded to be dilute enough at this density just below T_c. Furthermore, since the long-range interaction among glueballs is mediated by the virtual one-glueball exchange process in quenched QCD, the interactions among glueballs are exponentially suppressed beyond its Compton length $1/m_{G(S)} = 0.112$ fm. Therefore, one cannot expect the strong long-range interaction acting among the spatially separated thermal glueballs, and the dilute gas treatment of the glueballs is considered to be valid at least in the confinement phase in quenched QCD.

Now, it is quite difficult to imagine how such a too rare excitation of thermal glueballs can lead to the phase transition. We have thus arrived at the mystery. What is really the trigger or the driving force of the deconfinement phase transition in quenched QCD? Actually, also in full QCD, the trigger of the QCD phase transition would be an interesting question, but not so many have been known yet concerning this issue. In this sense, a careful consideration of this problem may provide us with a key to find out the trigger of the QCD phase transition.

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